Final Revision T&F / MCQ

Determine whether the statement is true or false:

1. Similar matrices have the same inverse. F

2. The $n \times n$ matrix A is invertible if and only if 0 is not an eigenvalue of A . T

3. If A and B are invertible similar matrices, then their inverses A^{-1} and $B^{-1}$are also similar. T

4. If A is an $m \times n$ complex matrix and B is an $n \times r$ complex matrix, then $\overline{AB} = \bar{A}\,\bar{B}$. T

5. Matrix C is diagonalizable if it is similar to a diagonal matrix B; there exists an invertible matrix P where $B = PCP^{-1}$. F

6. If Q is an orthogonal matrix, then $det\ (Q)$ must be 1. F

7. A square complex matrix A is called Unitary if its conjugate transpose equals its inverse. T

8. A linear transformation preserves the operations of vector addition and scalar multiplication. (True).

9. If the linear transformation $T : V \longrightarrow W$ is both one-to-one and onto, then it is an isomorphism. (True).

10. If we interchange two rows in an identity matrix, then it will not have an LU-decomposition. (True).

11. Every square matrix has an LU-decomposition. (False).

12. LU decompositions are unique. (False).

13. Simplex method is an iterative procedure for solving LPP in finite number of steps. (True).

14. The solution set of a system of linear equation is bounded if it can be enclosed by a circle. (True).

Final Revision T&F / MCQ

(a) If $T : V \to W$ is a one to one linear transformation, then $ker(T) = \{0\}$.

(a) True

(b) If $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a map given by $T(x, y) = (x + y, y - 1)$, then T is linear.

(b) False

(c) Every square matrix can be decomposed into LU-decomposition.

(c) False

(d) If A is $m \times n$ matrix, then the eigen values of $A^T A$ can not be negative.

(d) True

(e) The following L.P.P has an unbounded feasible region.

$$\begin{aligned} & \min \; z = x - y \\ \textit{subject to} \quad & 4x - 3y \geq 0 \\ & x + y \leq 10 \\ & x \geq 0, \quad y \geq 0. \end{aligned}$$

(e) False

(f) No L.P.P with an unbounded feasible region has a solution.

(f) False

Final Revision T&F / MCQ

(a) The function $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by $T(x_1, x_2) = (2x_1 + 3x_2, 4x_2 - 1 - x_1, x_1)$ is a linear transformation.

(a) False

(b) If $T : V \to W$ be an isomorphism, then $ker(T) = \{0\}$.

(b) True

(c) Every square matrix has a LU-decomposition.

(c) False

(d) If A is an $m \times n$ matrix, then $A^T A$ is an $m \times m$ matrix.

(d) False

(e) In linear programming problems, all variables are restricted to positive values only.

(e) False

(f) One of the quickest ways to plot a constraint is to find the two points where the constraint crosses the axes, and draw a straight line between these points.

(f) True

Final Revision T&F / MCQ

Linear Algebra (Math 251)
Level IV, Assignment 4
(2015-16)

1. State whether the following statements are true or false:

   (a) If a linear transformation T is an isomorphism, then kernel of T is the zero subspace.

   (a) True

   (b) If T is a translation operator, than it is linear.

   (b) False

   (c) Every square matrix have LU-factorization.

   (c) False

   (d) $\begin{pmatrix} 4 & 0 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 7 & 3 \\ 0 & -2 \end{pmatrix}$ is an LU-factorization of $\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$.

   (d) False

   (e) In linear programming problems, a linear objective function that is to be maximized or minimized.

   (e) True

   (f) The graphical method is used only when the LPP have exactly two unknown variables.

   (f) True

2. Select one of the alternatives from the following questions as your answer.

   (a) Let $T : \mathbb{R}^6 \to \mathbb{R}^5$ be a linear transformation with rank 4, then the number of basis element in the kernel of T is

   A. 2
   B. 4
   C. 6
   D. 10

   (b) If $T : V \to V$ be a linear operator such that $T(u) = 4,\ \forall u \in V$, then

A. T is linear.

B. T is isomorphism.

C. T is not linear.

D. None of the above.

(c) Which of the following sets of eigen values have a dominant eigen value

A. $\{2, -3, 4, 5, -5, -4\}$

B. $\{1, 7, -6, 4, -7, 3\}$

C. $\{-10, 11, -17, 4, 10, -11\}$

(d) The singular values of the matrix $A = \begin{pmatrix} 4 & 0 \\ 0 & 0 \\ 3 & 5 \end{pmatrix}$

A. 10, 40

B. 15, 35

C. $\sqrt{15}$, $\sqrt{35}$

D. $\sqrt{10}$, $\sqrt{40}$

(e) The point (3,0) satisfy one of the following systems

A.

$$\begin{aligned} x + y &\geq 5 \\ x + 2y &\geq 3 \end{aligned}$$

B.

$$\begin{aligned} 3x - y &\geq 9 \\ 4x + 5y &\leq 11 \end{aligned}$$

C.

$$\begin{aligned} 12x - y &\geq 35 \\ 3x + 4y &\leq 10 \end{aligned}$$

D.

$$\begin{aligned} 2x + y &\geq 6 \\ 3x - 5y &\geq 15 \end{aligned}$$

(f) The valid objective function for a linear programming problem is:

A. $\max(xy)$

B. $\min(3x - 2y + \frac{1}{2}z)$

Final Revision T&F / MCQ

1. State whether the following statements are true or false:

   (a) The product of eigen values of a matrix is same as its determinants.

   (a) True

   (b) The eigen values of the matrix $A = \begin{pmatrix} 2 & 0 & 0 \\ 6 & -1 & 0 \\ 17 & 3 & 4 \end{pmatrix}$ are 2, 4 and 0.

   (b) False

   (c) The inner product of two vectors cannot be a negative real number

   (c) False

   (d) If $v = (3, 4)$ then $\|v\| = 5$.

   (d) True

   (e) In an inner product space $(V, <, >)$ if x and y are unit vectors orthogonal to each other then $\|x + y\| = 2$.

   (e) False

   (f) If $u = (4, 3, 1, -2)$ and $v = (-2, 1, 2, 3)$ then $< u, v >= -9$.

   (f) True

   (g) The matrix $A = \begin{bmatrix} 7 & 1-i & 8 \\ 1-i & 5 & -1-6i \\ 8 & 6i-1 & -1 \end{bmatrix}$ is Hermitian.

   (g) False

   (h) A square matrix A is orthogonal, if $A^{-1} = A^T$.

   (h) True

Final Revision T&F / MCQ

(a) If 2, 3 and 4 are eigen values of a matrix A, then $\det(A) = 9$.

(a) False

(b) (1,-1,2) is the real part of the complex vector $(1+i, -1+i, i, 2)$.

(b) False

(c) The inner product of a nonzero vector with itself is always a positive real number.

(c) True

(d) If $u = (1, -1)$, $v = (-2, 2)$ and $k = 4$, then $< ku, v >=\ 16$.

(d) False

(e) If determinant of a matrix is 1 or -1, then the matrix is orthogonal.

(e) False

(f) The rows and columns of an orthogonal matrix are orthonormal.

(f) True

(a) The sum of eigenvalues of a square matrix is same as its determinant.

(a) False

(b) (1,0,3) is the real part of the complex vector $(2i+1, -3i, i+3)$.

(b) True

(c) The inner product of a vector with itself can not be negative real number.

(c) True

(d) If u and v are orthogonal vector then the angle between these two vectors is zero.

(d) False

(e) If determinant of a matrix is 1 or -1, then the matrix is orthogonal.

(e) False

(f) In case of real matrices, Hermitian and symmetric matrices are same.

(f) True

Final Revision T&F / MCQ

For Each Question, Choose the Correct Answer from the Multiple-Choice List.

1. If u = $\langle 1,2 \rangle$, v = $\langle 1,0 \rangle$ and $w = \langle -1,2 \rangle$, then $\langle u+v, w \rangle$=
   a. 0
   b. 2
   c. $\langle -1,4 \rangle$

2. The quadratic form expressed in $\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ is:
   a. $2x^2 + y^2 + 4x - y$
   b. $3x^2 - 3y^2$
   c. $2x^2 - y^2 + 5xy$

This should be: $\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & 5/2 \\ 5/2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$, as the matrix in the quadratic form must be symmetric.

3. Let $V = \begin{bmatrix} i \\ -1 \\ -i \end{bmatrix}$ a complex matrix, then $\bar{\bar{V}}$:

   a. $\begin{bmatrix} -i \\ -1 \\ i \end{bmatrix}$
   b. $\begin{bmatrix} i \\ -1 \\ -i \end{bmatrix}$
   c. $\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

Final Revision T&F / MCQ

4. One of the following matrices has no LU-decomposition

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 1 & -1 \\ -2 & -1 & 2 \\ 2 & 1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 1 & 2 \end{bmatrix}$

5. Let $T: U \longrightarrow V$ be a linear transformation, then:
   a. The kernel of T is a subspace of U
   b. The kernel of T is a subspace of V
   c. The range of T is a subspace of U
   d. None.

6. Graphical method can be applied to solve LPP when there are only:
   a. One variable.
   b. Two variables.
   c. Three variables.
   d. None.

7. Let $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be the multiplication by $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ then $T^{-1}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$will be equal to:
   a. $T^{-1}(x, y) = (x - 2y, -2x + 3/2y)$
   b. $T^{-1}(x, y) = (x - 1/2y, -2x + 5y)$
   c. $T^{-1}(x, y) = (x - 1/2y, -2x + 3/2y)$
   d. $T^{-1}(x, y) = (x - 1/2y, 2x + 3/2y)$

Final Revision T&F / MCQ

2. Select one of the alternatives from the following questions as your answer.

   (a) Which of the following sets of vectors are orthogonal with respect to the Euclidean inner product on R^2:

   A. (0,6), (7,0)
   B. (3,4),(2,6)
   C. (6,9),(5,2)
   D. (0,4), (0,6)

   (b) If $\|u\| = \sqrt{30}, \quad \|v\| = \sqrt{18}$ and $< u, v >= -9$, then $\cos\theta =$

   A. $\frac{-2}{3\sqrt{15}}$
   B. $\frac{-3}{2\sqrt{15}}$
   C. $\frac{-2}{3\sqrt{60}}$
   D. None

   (c) The values of k for which $u = (k, -4, 8)$ and $v = (k, k, -4)$ are orthogonal in $\mathbb{R}^3$ Euclidean Inner Product Space are

   A. 8, -4
   B. 4, -8
   C. -4, -8
   D. 4, 8

   (d) The eigen values of a Hermitian matrix are

   A. complex only
   B. complex and real both
   C. always zero
   D. always real

   (e) If 0 is an eigen value of a square matrix A then A is

   A. an Identity matrix.
   B. invertible.
   C. not invertible.
   D. None

   (f) If square matrix A is such that $AA^* = I$, then A is

   A. Hermitian
   B. Unitary
   C. skew-symmetric
   D. None

Final Revision T&F / MCQ

(g) The matrix $A = \begin{pmatrix} \frac{1}{9} & \frac{8}{9} & -\frac{4}{9} \\ \frac{4}{9} & -\frac{4}{9} & -\frac{7}{9} \\ \frac{8}{9} & \frac{1}{9} & \frac{4}{9} \end{pmatrix}$ is

A. Hermitian

B. Unitary

C. skew-symmetric

D. Orthogonal

3. Compute $< U, V >$ using the inner product on $M_{2\times 2}$, where

$$U = \begin{pmatrix} 9 & -8 \\ 9 & 18 \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} -1 & 9 \\ 1 & 1 \end{pmatrix}.$$

2. Select one of the alternatives from the following questions as your answer.

(a) If $u = (3, -1, 4)$, $v = (0, 4, 6)$ and $k = 2$, then the value of $< ku, v >=$

A. 20

B. -20

C. 40

D. -40

Final Revision T&F / MCQ

(b) If $p = 4 + 3x - 2x^2$ be a vector in the vector space P_2, then $\|P\| =$

A. $\sqrt{7}$
B. $\sqrt{21}$
C. 5
D. $\sqrt{29}$

(c) The eigenvalues of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 3 \\ 0 & 0 & 4 \end{bmatrix}$

A. $\{2, 3, 4\}$
B. $\{1, -1, 4\}$
C. $\{1, 0, -1\}$
D. $\{1, -1, 3\}$

(d) If $\{1, 4, -2\}$ be eigenvalues of a square matrix, then its determinant will be

A. -8
B. 3
C. 7
D. 8

(e) If $6x_1^2 + 3x_2^2 - 12x_1x_2$ be the quadratic form, then the associated symmetric matrix will be

A. $\begin{bmatrix} 6 & -6 \\ 3 & -6 \end{bmatrix}$
B. $\begin{bmatrix} 3 & -6 \\ -6 & 6 \end{bmatrix}$
C. $\begin{bmatrix} 6 & -12 \\ -12 & 3 \end{bmatrix}$
D. $\begin{bmatrix} 6 & -6 \\ -6 & 3 \end{bmatrix}$

(f) For which value of a and b, the matrix $\begin{bmatrix} 3 & 1+i & 2+6i \\ a & -1 & 2-4i \\ 2-6i & b & 1 \end{bmatrix}$ is Hermitian?

A. $a = 1 + i, \quad b = 2 - 4i$
B. $a = 1 + i, \quad b = 2 + 4i$
C. $a = 1 - i, \quad b = 2 + 4i$
D. $a = 1 - i, \quad b = 2 - 4i$

Final Revision T&F / MCQ

(a) The characteristic equation of the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$ is

A. $\lambda^2 - 7\lambda - 10 = 0$

B. $\lambda^2 + 7\lambda - 10 = 0$

C. $\lambda^2 - 7\lambda + 10 = 0$

D. $\lambda^2 + 7\lambda + 10 = 0$

(b) The eigenvalues of the matrix A^3, where $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 4 & 1 \\ 0 & 0 & -3 \end{bmatrix}$, are

A. $\{1, 4, -3\}$

B. $\{1, 12, -9\}$

C. $\{1, 64, 27\}$

D. $\{1, 64, -27\}$

(c) Which of the following sets of vectors are orthogonal with respect to the Euclidean inner product on $\mathbb{R}^2$:

A. (1,2), (-2,1)

B. (3,4),(2,6)

C. (6,9),(5,2)

D. (0,4), (0,6)

(d) If angle between vectors u and v is zero such that $\|u\| = 4$, $\|v\| = 6$, then $< u, v >=$

A. 10

B. 24

C. $\sqrt{24}$

D. $\sqrt{10}$

(e) If $3x_1^2 + 2x_2^2 - 4x_3^2 - 2x_1x_2 + 6x_1x_3 - 4x_2x_3$ be the quadratic form, then the associated symmetric matrix will be

A. $\begin{bmatrix} 3 & 1 & 3 \\ 1 & 2 & -2 \\ 3 & -2 & -4 \end{bmatrix}$

B. $\begin{bmatrix} 3 & -1 & -3 \\ -1 & 2 & -2 \\ -3 & -2 & -4 \end{bmatrix}$

C. $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 2 & 2 \\ 3 & 2 & -4 \end{bmatrix}$

Final Revision T&F / MCQ

D. $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 2 & -2 \\ 3 & -2 & -4 \end{bmatrix}$

(f) For which value of a and b, the matrix $\begin{bmatrix} 3 & i+2 & 2+6i \\ a & -1 & 2i-1 \\ 2-6i & b & 1 \end{bmatrix}$ is Hermitian?

A. $a = i-2, \quad b = -2i-1$

B. $a = -i+2, \quad b = 2i+1$

C. $a = -i+2, \quad b = -2i-1$

D. $a = i-2, \quad b = 2i+1$

Final Revision T&F / MCQ

(a) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear operator given by $T(x_1, x_2) = (x_2 - x_1, -2x_1 + 2x_2)$. Which of the following vector is in $Ker\ T$?

A. (-1,2)
B. (-1,1)
C. (1,-1)
D. (1,1)

(b) If $T : M_{44} \to \mathbb{R}^{10}$ be a linear transformation with rank 8, then nullity of T is given by

A. 8
B. 2
C. 4
D. 10

(c) Which of the following sets of eigenvalues have a dominant eigenvalue:

A. $\{6, -4, -6, 1\}$
B. $\{-3, -1, 0, 2\}$
C. $\{-10, 0, 1, 10\}$
D. None of the above

(d) If $B = \begin{bmatrix} 7 & 0 \\ 0 & 2 \end{bmatrix}$ be a matrix where $B = A^T A$, then the singular values of A are

A. $\{7, 0\}$
B. $\{0, 2\}$
C. $\{7, 2\}$
D. $\{\sqrt{7}, \sqrt{2}\}$

(e) In maximization problem, optimal solution occurring at corner point yields the

A. mean values of z
B. lowest value of z
C. mid values of z
D. highest value of z

(f) Which of the following constraints is not linear?

A. $7A - 6B \leq 45$
B. $X + Y + 3Z \geq 35$
C. $2XY + X = 15$
D. None of the above.

Final Revision T&F / MCQ

. Select one of the alternatives from the following questions as your answer.

(a) If $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear operator given by $T(x, y) = (2x - y, -4x + 2y)$, then which of the following vector is in $ker(T)$?

A. $(1, 4)$

B. $(2, 1)$

C. $(1, 1/2)$

D. $(1/2, 1)$

(b) If $T : W \to V$ be a linear transformation, then $ker(T)$ and $range(T)$ are subspaces of vector space(s)

A. V.

B. W.

C. W and V respectively.

D. V and W respectively.

(c) Which of the following sets of eigenvalues have a dominant eigenvalue:

A. $\{8, -7, -6, 8\}$

B. $\{-5, -2, 2, 4\}$

C. $\{-3, -2, -1, , 0, 1, 2, 3\}$

D. None of the above

(d) If $B = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 16 \end{bmatrix}$ be a matrix where $B = A^T A$, then the singular values of A are

A. $\{4, 9, 0\}$

B. $\{0, 9, 16\}$

C. $\{4, 9, 16\}$

D. $\{2, 3, 4\}$

(e) In linear programming, objective function and objective constraints are

A. solved.

B. quadratic.

C. adjacent.

D. linear.

(f) The feasible region

A. is defined by the objective function.

B. is an area bounded by the collective constraints and represents all permissible combinations of the decision variables.

C. represents all values of each constraint.

D. may range over all positive or negative values of only one decision variable.